Introduction

I should begin by tracing the main events that led me to *Self-Similar Melodies*, to give you an idea of where this book is coming from and who is writing it.

My formal education was strictly musical, but I always liked logic and mathematics, and by now I have been thinking about logical progressions in music for almost 30 years. At first my main source of information was W.R. Ashby's *An Introduction to Cybernetics*, which I read with great enthusiasm in 1966, as a student, and which provided the basis of a paper I wrote the following year called "Music as Machine." I wasn't able to go much further in this direction though, either musically or theoretically, until 1979 when someone gave me Douglas Hofstadter's *Gödel Escher Bach* for Christmas. Around that same time I also found Benoit B. Mandelbrot's *Fractals*, and Martin Gardner's *Mathematical Games* columns in *Scientific American*, and met David Feldman, a mathematician-composer who helped me understand what I was reading.

With this information I was able to make some important breakthroughs in my composing, especially with *Nine Bells* (1979) and the *Rational Melodies* (completed in 1982), though much of my music, such as *Riemannoper*, was relatively uninfluenced by logical sequences. Beginning in 1987, when I wanted to write mathematical *Music for 88*, I went back to mathematics once again. I read Euclid's number theory, Euler's *Mathematische Musik*, and some books on the history of mathematics, and followed a series of stimulating but difficult lectures in number theory given by Michel Waldschmidt at the Université de Paris VI. Then I met the mathematician Jean-Paul Allouche, who helped me a great deal, and continues to do so, and I read *Chaos and Fractals* by Heinz-Otto Peitgen, Hartmut Jürgens and Dietmar Saupe, which turned out to be a gold mine of information about mathematical systems, many of which related to my musical systems. Also important has been Paul Epstein, a composer and colleague in Philadelphia. Our correspondence since 1990 has been especially useful regarding self-replicating melodies.

During all this time I was gathering notes and charts and computer programs, but I was always busy trying to produce pieces of music, and I never wanted to take the time to try to explain the techniques I was using. Eventually, however, my files of such things were overflowing, and I wasn't even sure myself anymore what I had been doing and what I should do next. It was time to clean out my files–and my head. As I began to go through piles of forgotten paper, it seemed that it would be good to try to put together a summary of this information. Not only was it necessary to put my own thoughts in order, but much of what I had developed seemed potentially useful for other composers and music theorists, and perhaps also for some non-musicians interested in logical systems. In order to put all these ideas together in a coherent way that you can hopefully understand, I needed to limit my subject, and it seemed to me that the majority of the most interesting ideas I had been exploring could all be defined as "self-similar melodies," a term which I had best define before I go any further.

To define "self-similar" in all its mathematical senses would require going into Cantor staircases, Sierpinsky sponges, Hilbert curves, Peano curves, and many other subjects. Numerous other books already do all this, and there is no need for a musician like myself, with only a limited understanding of the subject, to attempt to do it again. Besides, melodies are basically one-dimensional, whereas most of these mathematically studied types of self-similar structures are two- and three-dimensional. The principle of a self-similar structure is the same in all cases, however: a structure that replicates itself on more than one level. A self-similar melody, then, is a melody in which the detailed movement from note to note is reflected in the way the melody as a whole is structured, and the larger organization reflects the details. This is not very precise language, so I will also give you an exact definition, which I can adhere to throughout this study:

> *Self-similar melodies are melodies constructed entirely by repeated applications of a single procedure.*

Most of the sequences of notes and numbers in this book have exactly the same organization embedded on at least two levels in a way that is clear and indisputable, but there are a few cases where one could dispute whether the self-similarity is really exact, so that is why I have made my definition somewhat cautiously. Of course, if a single procedure is repeated many times, there will necessarily be some relationship not only between the second, third, and fourth outcomes of this procedure, but also between the second, fourth and sixth outcomes, and between the third, ninth and 27th outcomes, and so on. So my definition is not far from those of Mandelbrot, Peitgen, and of current mathematicians in general.

Self-similarity may seem like something that has very little to do with music, but in fact it goes to the very center of what composition has been about in the 20th century, and much of the 19th century as well. I once asked John Cage, whom I had the good fortune to know fairly well, what he really learned from his teacher Arnold Schoenberg. Usually Cage responded to this question with the often published anecdote about how Schoenberg said he would never have a sense of harmony. But this time, speaking informally late in his life, he replied differently. "I think the most important thing I learned from him was that the microcosm and the macrocosm should be related. Schoenberg talked often about that, and started me thinking in this direction."

The relationship is rather easy to see in the music of Cage, where chance determines what happens on every level, from the overall form to the individual note, and it is not difficult to understand how Schoenberg must have thought about the relationship between his angular expressionistic instrumental lines and the angular expressionistic forms that contain them. It should be pointed out too that both Cage and Schoenberg were no doubt familiar with the theoretical studies of Heinrich Schenker, who demonstrated in detail how the individual harmonies and melodic motions reflect the forms and modulation schemes in the great German tradition from Bach to Brahms. Schenker thought that 20th-century music had nothing to do with his studies of musical "foreground," "middleground," and "background," and it is true that such relationships are difficult to find in Stravinsky, Bartok, Berio, and in fact, the majority of 20th-century music. But foreground-background relationships are very clear in the music of Schoenberg and Cage, though they did it in ways quite different from one another, and from Beethoven. The more precisely calculated microcosm-macrocosm relationships that you shall find in the melodies of this book are simply other manifestations of the same principle.

Limiting my subject in this way has of course required eliminating many things. Combinations and permutations, for example, have been especially useful to me as a compositional tool. How many melodies can be constructed by passing through a single grid or labyrinth? How many melodies of eight notes can one construct using only thirds and fourths? Such questions have sometimes been very useful in composing, but they are not really relevant to self-similar structure. I also had to forget techniques having to do with special musical materials such as glissandos, and eliminate everything having to do with counterpoint, orchestration and other non-melodic aspects of music.

In addition, I decided to exclude analysis of my own compositions. Actual compositions are full of texts and contexts, instrumentation and interpretation questions, humorous notes and satiric notes, ethical and political messages, and all sorts of things that are difficult to explain and have little or nothing to do with the behavior of self-similar melodies. So the examples in this book were all written specifically to demonstrate whatever they demonstrate and are not intended to have any particular musical value outside this context. At the end of each section, references are given to actual compositions using the techniques discussed, for those of you who get tired of looking at the short examples and want to see what these things might sound like in real pieces of music.

To abstract the musical examples, and to make them relevant in a more general sort of way, I have written most of them with percussion clef, simply indicating higher and lower scale degrees. This gets us around the problem of whether the particular example might sound better in the upper register or the lower register, in one mode or another, on a European scale or in some other tuning system, and other such considerations having nothing to do with self-similar melodic structure. And very often, when I just wanted to show a short sequence of notes growing into a longer melody, I haven't used musical symbols at all, but simply numbers.

One more thing before we get started. This book is not about esthetics. I am going to simply assume that you are sympathetic with what I would call a rationalist or structuralist approach to art and music, and I won't attempt to convince you that this approach is necessarily better than any of the romantic, expressionistic, intuitive, mystical and otherwise subjective approaches to art and music. But a brief summary of the structuralist point of view should help you to orient yourself to the topography of this book, and see how you want to move around in it.

The structuralist supposes that conscious awareness is more interesting than dreams, that it is better to know what one is doing than not to know, and that objective reality is more interesting than subjective experience. The structuralist prefers to think about up and down, single and double, greater and smaller, things that are not limited to particular cultures and particular periods of history. The structuralist tries to work with elements that can be defined and measured, and tries, as much as possible, not to think about things that can not be defined and measured, such as fear, beauty, perception, boredom, and inspiration.

But having written that paragraph, I must quickly point out that many questions remain: Why is it, even after something has been rigorously proved, that it can sometimes continue to seem ultimately mysterious? What are my motivations for doing what I do? Why do I find that one descending melody thrills me while another descending melody, written with exactly the same self-similar procedure, leaves me completely cold? Why is it that I have so much trouble going to sleep when I know there is an uncorrected error in some score I am working on? Are we talking about detached logic or total compulsiveness?

You've probably heard many times terms like "cold facts" and "hard logic," but rationality is neither hard nor cold when it is done by soft warm-blooded animals like ourselves.

References

Allouche, Jean-Paul and Johnson, Tom: “Finite Automata and Morphisms in Assisted Musical Composition” (1994), published by the Laboratoire de mathématiques discrètes, Centre national de la recherche scientifique, Marseille. This short collaborative study is primarily a mathematical analysis of the formulas used in my *Formulas for String Quartet* (1994), but it also provides an introduction to automata in general, and to how they can be applied to music.

Ashby, W. R.: *An Introduction to Cybernetics*, (1963), Wiley, New York. When computer science was still young and the principles of artificial intelligence were still new, the discussion was on a rather theoretical level, and I came away with a new understand of things like “switches” and “transformations.” I suspect the book is still stimulating for many readers today.

Hofstadter, Douglas: *Gödel Escher Bach* (1979), Basic Books. This layman’s guide to Gödel also taught me a lot about logical sequences, recursive functions, and many other things. It was a very popular book in America at this time.

Gardner, Martin: The “Mathematical Games” columns, which Gardner wrote for many years in *Scientific American*, contain hundreds of ideas that can be useful for musicians and artists, but the most important for me personally were the explanations of the “Dragon” formula in several installments in 1967.

Johnson, Tom: I’ll give references later to specific compositions having to do with specific phenomena, but three scores should be cited already, since they are particularly important in the general evolution of my “self-similar melodies."

Nine Bells (1979), Editions 75, is a 50-minute dance/ performance played on nine suspended bells. It is one of my first pieces that really follows logical sequences, though the logic is more geometric than numeric.

Rational Melodies (completed in 1982), Editions 75, is a collection of 21 melodies for any instrument, with explanations of the processes used to compose them. Also available as a CD (Hat Art 613).

Music for 88 (1988), Editions 75. Nine separate pieces including the *Mersenne Numbers, the Multiplication Table, Abundant numbers, Square Numbers, Eratosthenes’ Sieve, Euler’s Harmonies, and Pascal’s Triangle*. Also available as a CD (Experimental Intermedia XI 106).

All Editions 75 publications are available in America through the Two-Eighteen Press (5004 Applewood Circle, Carmel, NY 10512-2640).

Mandelbrot, Benoit B.: *Fractals: Form, Chance and Dimension* (1977), W. H. Freeman and Co. A lot of the mathematics went over my head when I first read this book, and some of it still does, but I could look at the pictures and see what Mandelbrot was talking about, and I learned a lot about self-similar structure.

Peitgen, Heinz-Otto, Jürgens, Hartmut and Saupe, Dietmar: *Chaos and Fractals* (1st edition 1992, 2nd edition 2004), Kaiser Verlag. A big book, over 900 pages long, that assembles mountains of recently discovered information in a way that non-mathematicians can understand. The second chapter (pp. 63-134) provides a very good overview of "Classical Fractals and Self-Similarity." Later I'll be making use of specific ideas from this book, such as the MRCM (multiple reduction copying machine), and repeating some of the observations on Pascal's triangle and other automata. These mathematicians, and their colleagues at the Centrum für complexe Systeme und Visualisierung in Bremen, have produced a number of books, often with very attractive computer graphics, but this is the one I read, and perhaps the most complete at this point.